MATH 211 EXAM ONE
Fall 2005
September 29

Name: ______________________________

The test consists of 2 pages. Justify your work when necessary.

(1) Find a simplifying expression for the following sets. Here $\mathcal{U}$ is the universe, A and B are two sets such that $B \subseteq A$.

a) $A \cup \emptyset$	b) $A \cup \mathcal{U}$
c) $A - \mathcal{U}$	d) $A \oplus A$
e) $\emptyset - A$	f) $A \cap B$
g) $A \cup B$	h) $\overline{A} \cap B$
i) $A \oplus B$	j) $\mathcal{P}(\emptyset)$

(2) Using only p, q, r, $\neg$ and/or the connective $\wedge$, write a proposition equivalent to each of the following

(a) $(p \to q) \to r$

(b) $p \to (q \to r)$

(3) Write the contrapositive and converse of the statement: "You sleep late if it is Saturday".

(4) In the following, $P(x, y)$ means "$x + 2y = xy$". Where x and y are integers. Determine the truth value of the statement.

(a) **T** **F** $\exists y P(x, 3)$

(b) **T** **F** $\forall x \exists y P(x, y)$

(c) **T** **F** $\exists x \forall y P(x, y)$

(5) Suppose the variable x represents students and the variable y represents courses, and $A(y)$: y is an advanced course $S(x)$: x is a sophomore $F(x)$: x is a freshman $T(x, y)$: x is taking y. Write the following statements using these predicates and any needed quantifiers.

(a) There is a course that every freshman is taking.

(b) No freshman is a sophomore.

(c) Some freshman is taking an advanced course.

(d) There are at least two freshman students taking the exact same courses.

(6) Determine whether the following argument is valid.

$p \to r$
$q \to r$
$\underline{\neg(p \vee q)}$
$\therefore \neg r$

(7) Determine whether the following argument is valid.
She is a Math Major or a Computer Science Major.
If she does not know discrete math, she is not a Math Major.
If she knows discrete math, she is smart.
She is not a Computer Science Major.
Therefore, she is smart.

(8) Determine whether the rule describes a function. If your answer is no say why.

(a) $f : \mathbb{N} \to \mathbb{N}$ where $f(n) = \sqrt{n}$.

(b) $g : \mathbb{N} \to \mathbb{N}$ where $g(n) =$ any integer $> n$.

(9) Give an example of a function from $\mathbb{Z}$ to $\mathbb{N}$ that is both one-to-one and onto.

(10) Give an example of a function from $\mathbb{Z}$ to $\mathbb{N}$ that is onto but NOT one-to-one.

(11) Let $f : A \to B$. Let $B' \subset B$. Show that $f(f^{-1}(B')) \subseteq B'$. WHAT condition is needed for the containment in the other direction?